

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
A A A A A A	3. RECIPIENT'S CATALOG NUMBER
15194.12-M	<b>p</b> 3
TITLE (and Subtitio)	S TYPE OF REPORT & PERIOD COVERED
(12)	Final:
Multicharacteristic Quality Control	1 September 1978 - 31 Aug 81
AUTHOR(e)	8. CONTRACT OR GRANT NUMBER(+)
D. R. Jensen	DAAG29 78 G 0172
P. M. Ghare	
PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Virginia Polytechnic Institute & State University	li .
Blacksburg, VA	
CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
U. S. Army Research Office	Feb 82
Post Office Box 12211	13 NUMBER OF PAGES
Research Triangle Park, NC 27709	5
MONITORING AGENCY NAME & ADDRESS(II different from Controlling Office)	15. SECURITY CLASS, (of this report)
	Unclassified
	ISA DECLASSIFICATION/DOWNGRADING
	SCHEDULE
6. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlin	
Approved for public release; distribution unlin	ni ted.
Approved for public release; distribution unlin	ni ted.
Approved for public release; distribution unlin	ni ted.
Approved for public release; distribution unlin	ni ted.
Approved for public release; distribution unlin  17. DISTRIBUTION STATEMENT (of the ebstract entered in Black 20, if different f	ni ted.
Approved for public release; distribution unlin  7. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, If different for the abstract entered in Black	nited.  From Report)  MAR 1 8 1982
Approved for public release; distribution unling.  7. DISTRIBUTION STATEMENT (of the abetract entered in Black 20, 11 different for the supplementary notes. The view, opinions, and/or findings contained in the supplementary notes.	nited.  NAR 18 1982  In this report are those of the
Approved for public release; distribution unlin 7. DISTRIBUTION STATEMENT (of the abetract entered in Black 20, if different f NA	n this report are those of the icial Department of the Army
Approved for public release; distribution unling.  7. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, 11 different for the supplementary notes.  The view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design	n this report are those of the icial Department of the Army lated by other documentation.
Approved for public release; distribution unline.  7. DISTRIBUTION STATEMENT (at the obstract entered in Black 20, if different for the supplementary notes.  The view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design.  19. KEY WORDS (Continue on reverse side II necessary and identity by block numbers.)	n this report are those of the icial Department of the Army lated by other documentation.
Approved for public release; distribution unline.  7. DISTRIBUTION STATEMENT (at the abstract entered in Black 20, if different for the view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design quality control forecasting	n this report are those of the icial Department of the Army lated by other documentation.
Approved for public release; distribution unline.  7. DISTRIBUTION STATEMENT (at the obstract entered in Black 20, if different for the supplementary notes.  The view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design.  19. KEY WORDS (Continue on reverse side II necessary and identity by block numbers.)	n this report are those of the icial Department of the Army lated by other documentation.
Approved for public release; distribution unline.  7. DISTRIBUTION STATEMENT (of the obstract entered in Black 20, 11 different for the view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design quality control forecasting diagnostic procedures	n this report are those of the icial Department of the Army lated by other documentation.
Approved for public release; distribution unline.  7. DISTRIBUTION STATEMENT (at the abstract entered in Black 20, if different in the supplementary notes.  The view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design quality control forecasting diagnostic procedures process monitoring industrial processes.	n this report are those of the licial Department of the Army lated by other documentation.
17. DISTRIBUTION STATEMENT (of the obstract entered in Black 20, If different in the supplementary notes  The view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design quality control forecasting diagnostic procedures process monitoring industrial processes	n this report are those of the icial Department of the Army ated by other documentation.
Approved for public release; distribution unline  17. DISTRIBUTION STATEMENT (of the abetrect entered in Black 20, 11 different for the view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design quality control forecasting diagnostic procedures process monitoring industrial processes	n this report are those of the icial Department of the Army ated by other documentation.
Approved for public release; distribution unline  17. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, if different for the supplementary notes.  The view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design quality control forecasting diagnostic procedures process monitoring industrial processes.  ABSTRACT (Continue on reverse of the first energy and identify by block number Results are presented for research whose purpose procedures for use when several quality characte item. Special emphasis was to be given to devel	n this report are those of the icial Department of the Army ated by other documentation.  was to develop quality control ristics are observed for each oping diagnostic procedures
Approved for public release; distribution unline  17. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, if different for the view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design quality control forecasting diagnostic procedures process monitoring industrial processes  ABSTRACT (Continue on reverse side if necessary and identify by block number of the process of the proc	n this report are those of the icial Department of the Army ated by other documentation.  was to develop quality control ristics are observed for each oping diagnostic procedures
Approved for public release; distribution unline  17. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, if different for the view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design quality control forecasting diagnostic procedures process monitoring industrial processes  20. ABSTRACT (Continue on reverse side if necessary and identify by block number of the procedures of the process of t	n this report are those of the icial Department of the Army ated by other documentation.  was to develop quality control ristics are observed for each oping diagnostic procedures

DD , FORM 1473 EDITION OF ! NOV 65 IS OBSOLETE

UNCLASSIFIED

## FINAL REPORT

- 1. ARO PROPOSAL NUMBER: 15194-M
- 2. PERIOD COVERED BY REPORT: September 1, 1978 -- December 15, 1981
- 3. TITLE OF PROPOSAL: "MULTICHARACTERISTIC QUALITY CONTROL"
- 4. GRANT NUMBER: DAAG-29-78-G-0172
- 5. NAME OF INSTITUTION: VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
- 6. AUTHORS OF REPORT: D.R. JENSEN and P.M. GHARE
- 7. PARTICIPATING SCIENTIFIC PERSONNEL

DEPARTMENT OF STATISTICS

DEPARTMENT OF INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH

## PRINCIPAL INVESTIGATORS

Dr. D.R. Jensen

Dr. M.R. Reynolds

Dr. P.M. Ghare

# SUPPORTING PERSONNEL AND DEGREES AWARDED

Mr. Y.V. Hui

(Ph.D. August 29, 1980.)

Mr. S.J. Sen

(Ph.D. unrelated to project.)

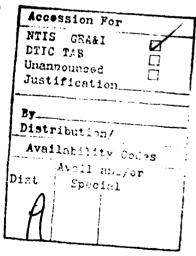
Mr. Mark O. Marcucci

(Ph.D. expected Summer 1982.)

Mr. J.A. Creiman (M.S. expected Summer 1982.)

- 8. SUMMARY OF THE PRINCIPAL FINDINGS (See attached pages.)
- LIST OF PUBLICATIONS AND REPORTS (See attached pages.)
- 10. LIST OF PAPERS PRESENTED AND WORKSHOPS ORGANIZED (See attached pages.)
- 11. CONTINUING ACTIVITIES (See attached pages.)





#### 8. SUMMARY OF THE PRINCIPAL FINDINGS

#### A. OBJECTIVES

The purpose was to develop quality control procedures for use when several quality characteristics are observed for each item. Special emphasis was to be given to developing diagnostic procedures for assigning causes when product quality is unacceptable.

## B. THE PRINCIPAL RESULTS

These findings have appeared in published and unpublished reports, or in papers presented at professional meetings, or in materials presented to Workshops and Short Courses, or they are forthcoming in a Ph.D. Dissertation now in preparation.

- 1. Six different versions of Hotelling's T<sup>2</sup> charts were developed for use in monitoring process means in a variety of circumstances. Diagnostic procedures were devised for use with each type of T<sup>2</sup> chart.
- 2. Control charts for monitoring process dispersion parameters were developed, along with diagnostic procedures for use with these charts.
- Control charts and diagnostics were developed for monitoring both variances in the bicharacteristic case.
- 4. Sequential acceptance procedures were developed for use when the failures follow a Weibull distribution.
- 5. Procedures were considered for monitoring the stationarity of models used in forecasting.
- 6. Disturbances in  $\overline{X}$ , R, and S<sup>2</sup> charts were studied owing to the use of estimates from a base period. Stochastic bounds were developed for distributions of run lengths associated with these charts.
- 7. Run lengths of the R, S<sup>2</sup>, and T<sup>2</sup> charts were studied when the monitored processes are nonstationary. Stochastic bounds for the actual run lengths were developed in terms of run-length distributions appropriate for stationary processes.
- 8. A basic probability inequality for the joint distribution of  $T^2$  statistics was developed. This result has applications in assessing the performance of  $T^2$  charts under nonstandard conditions.
- 9. Multivariate inequalities of Chebychev's type were developed. An application yields a universal distribution-free stochastic found for the run lengths of S<sup>2</sup> charts for any underlying distribution have found moments.
- 10. The notion of an optimal control chart was developed in terms of properties of run lengths. The S<sup>2</sup> and T<sup>2</sup> charts were shown to be optimal among large classes of procedures in the sense that their run lengths are stochastically smallest when the respective processes are not in control.
- 11. A general model for monitoring a process was formulated using several levels of unacceptability having diagnostic value. Properties of the Markov process

- governing the model were used to find means and variances of the number of interruptions and the numbers of adjustments of various types made to the process.
- 12. Charts were developed for monitoring the stationarity of the form of an input-output model for production, and for monitoring the residual variance of the process.
- 9. LIST OF PUBLICATIONS AND REPORTS
- A. PAPERS PUBLISHED AND ACCEPTED
- 1. Jensen, D.R. (1980). "Bounds on the Joint Distribution of  $T_0^2$  Statistics." Ann. Inst. Statist. Math. 32, 37-42.
- 2. Jensen, D.R. (1980). "Inequalities for Estimative and Predictive Probabilities." Biometrical J. 22, 739-743.
- 3. Ghare, P.M. (1981). "Sequential Tests under Weibull Distribution." Proc. 1981 IEEE Symposium on Reliability and Maintainability, pp.375-380.
- 4. Ghosh, B.K., Reynolds, M.R. and Hui, Y.V. (1981). "Shewhart X-Charts with Estimated Process Variance." Commun. Statist. -- Theor. Meth. Al0, 1797-1822.
- 5. Reynolds, M.R. and Ghosh, B.K. (1981). "Designing Control Charts for Means and Variances." 1981 ASQC Quality Congress Trans., pp.400-407.
- 6. Marcucci, M. and Jensen, D.R. (1981). "Nonstandard Distributions in Two-Stage Least Squares." Proc. 1981 Business and Economic Statist. Sec., ASA, pp.471-474.
- 7. Jensen, D.R. (1982). "Inequalities for Joint Distributions of Quadratic Forms." SIAM J. Appl. Math. 42 (To appear.)
- 8. Jensen, D.R. (1982). "An Optimal Property of S<sup>2</sup> Charts." J. Statist.
  Comput. Simul. 12 (To appear.)

## B. DISSERTATION

Hui, Y.V. (1980). Topics in Statistical Process Control. Ph.D. Dissertation, Library, VPI&SU, Blacksburg, Virginia.

# C. TECHNICAL REPORTS

Number	Authors, Title and Date
Q-1	Jensen, D.R. "Inequalities for Joint Distributions of Quadratic Forms." December 1979.
Q-2	Ghare, P.M., Hui, Y.V. and Jensen, D.R. "Multicharac- teristic Quality Control: A Survey." March 1980.
Q-3	Jensen, D.R. and Hui, Y.V. "The Use of R and S <sup>2</sup> Charts under Nonstandard Conditions." March 1980.
Q-4	Ghosh, B.K., Reynolds, M.R. and Hui, Y.V. "Shewhart $\overline{X}$ -Charts with Estimated Process Variance." March 1980.
Q-5	Hui, Y.V. and Jensen, D.R. "Markovian Time-Delay Sampling Policies." August 1980.
Q-6	Jensen, D.R., Hui, Y.V. and Ghare, P.M. "Monitoring an Input-Output Model for Production. I. The Control Charts." December 1980.
Q-7	Ghare, P.M. "Sequential Tests under Weibull Distribution." December 1980.
Q-8	Jensen, D.R. "Invariant Ordering and Order Preservation." May 1981.
Q-9	Marcucci, M. and Jensen, D.R. "Nonstandard Distributions in Two-Stage Least Squares." May 1981.
Q-10	Jensen, D.R. and Hui, Y.V. "Properties of T <sup>2</sup> Charts in Monitoring Process Means." June 1981.
Q-11	Jensen, D.R. "An Optimal Property of S <sup>2</sup> Charts." June 1981
Q-12	Ghare, P.M. "Monitoring and Revalidation of Box-Jenkins Type Forecasting Models." March 1982.

## D. PAPERS SUBMITTED FOR PUBLICATION

- Hui, Y.V. and Jensen, D.R. "Markovian Time-Delay Sampling Policies", submitted to *Technometrics*.
- Jensen, D.R., Hui, Y.V. and Ghare, P.M. "Monitoring an Input-Output Model for Production. I. The Control Charts." Submitted to Management Science.
- Jensen, D.R. and Hui, Y.V. "The Use of R and  ${\rm S}^2$  Charts under Nonstandard Conditions."

## 10. LIST OF PAPERS PRESENTED AND WORKSHOPS ORGANIZED

## A. PAPERS PRESENTED

- "Shewhart X-Charts with Estimated Process Variance," by B.K. Ghosh, M.R. Reynolds and Y.V Hui, to the Annual Meeting of the American Statistical Association, Houston, August 1980.
- "Sequential Tests under Weibull Distribution," by P.M. Chare, to the IFEE Reliability and Maintainability Symposium, Philadelphia, January 1981.
- "Monitoring and Revalidation of Box-Jenkins Type Forecasting Models," by P.M. Ghare, to the *International Symposium on Forecasting*, Quebec City, May 1981.
- "The Comparison of Several Multinomial Distributions with a Standard," by M.O. Marcucci and D.R. Jensen to the Virginia Academy of Science, Norfolk, May 1981.
- "Designing Control Charts for Means and Variances," by M.R. Reynolds and B.K. Ghosh, to the Annual ASQC Quality Congress and Exposition, San Francisco, May 1981.
- "Nonstandard Distributions in Two-Stage Least Squares," by M.O. Marcucci and D.R. Jensen, to the Business and Economic Section American Statistical Association, Detroit, August 1981.

# B. WORKSHOPS ORGANIZED

- "Multicharacteristic Quality Control," by P.M. Ghare and D.R. Jensen, for research personnel at the Picatinny Arsenal, Dover, NJ, July 28-29, 1981.
- "Statistical Procedures for Process Monitoring and Control," by P.M. Ghare and D.R. Jensen, the Annual Short Course preceding the Army Design Conference, Raleigh, NC, October 19-20, 1981.

## 11. CONTINUING ACTIVITIES

Work initiated under this project is continuing along several lines as follows.

Mr. Mark O. Marcucci is continuing work supported earlier toward a Ph.D. Dissertation in Statistics. The main focus of this work is process monitoring by attributes. He has developed generalized p-charts for the case of polychotomous responses. He is developing bivariate and multivariate versions of the p-charts and c-charts having diagnostic capabilities, using appropriate multidimensional distribution theory.

Mr. Ziv Barlach is undertaking studies toward a Fh.D. Dissertation in Industrial Engineering and Operations Research, in continuation of a project begun under ARO support. Specifically, his work centers on the problem of determining when an intervention has occurred in a time series, when using a forecasting model of Box-Jenkins type.